A FUZZY APPROACH TO NATURAL RESOURCE MANAGEMENT FROM A REGIONAL PERSPECTIVE

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ABSTRACT

Describes an approach to regional natural resource management that explicitly incorporates both the uncertainty and the multi-objective nature of such large-scale systems. A fuzzy linear programming algorithm is used to analyze sets of land management alternatives designed for individual forests within a region. Fuzziness is introduced in recognition that regional goal aspiration levels, interactions among resources, and cost estimates are imprecise and uncertain. An illustrative example, based on the 1989 RPA National Assessment Resource Interactions Model of the USDA Forest Service, is applied to the California Region and is used to test various fuzzy modelling approaches. Results show that: (a) regional optimization produces significant cost savings when compared with the solution obtained by aggregating individually-optimized forest plans for the region, and (b) fuzzy-based regional solutions may offer decision makers a wider range of preferable solutions than those based solely on linear programming.

KEYWORDS: Fuzzy linear programming, optimization, multi-resource planning, multi-objective analysis.

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INTRODUCTION

It is well documented in the forest management literature that the aggregation of decisions optimized at the sub-forest level (i.e., at the individual stand, drainage basin, or landscape level) leads to sub-optimization, and perhaps infeasibility, when compared to the optimization of decisions performed at the forest level. Similar results typically occur when decisions optimized at the individual forest level are aggregated at the regional level. And, the same can be said when regional decisions are aggregated at the state or national level. Sub-optimization occurs primarily because certain subunits (forests) are more proficient at meeting forest (regional) goals, but such efficiencies are lost when optimization is carried out at the lower levels of the organizational hierarchy. Infeasibilities can occur when constraints, omitted at the subunit level, are added to the decision matrix at a higher level of analysis.

One government agency faced with these issues is the USDA Forest Service, which began to recognize that these were serious problems in the 1970's. As a result, Congress passed the Forest and Rangeland Renewable Resources Planning Act of 1974 (RPA) and the National Forest Management Act of 1976 (NFMA). The RPA requires that the Forest Service implement coordinated planning across three administrative levels: national, regional, and forest. National planning is mandated to occur every ten years and must consider the costs and relative values of both market and non-market outputs from national forest lands. A portion of each national output target is allocated to each of the Forest Service regions and is incorporated into a regional guide. These guides establish standards and guidelines for addressing major issues and concerns at the regional level. The guides also provide tentative resource objectives for each forest within the region. Each forest must prepare a forest plan which adheres to the principles of multiple use and sustained yield of renewable resources without impairment of the land's productivity. Such plans are expected to maximize long term net public benefits in an environmentally sound manner. A range of alternatives which emphasize different resource objectives are formulated and evaluated. One alternative must correspond to the RPA-derived output targets assigned by the regional guide. Through a comprehensive, systematic, interdisciplinary approach, with early and frequent public participation, the adopted forest plan may lead to a modification of the tentative RPA targets.

The RPA planning process has characteristics of both a top-down and a bottom-up approach for coordinating resource use, allocating budgets, and setting and satisfying output targets among the three administrative levels. To date, however, there has been a limited amount of attention focused on the tradeoffs inherent in optimizing decisions at these various administrative levels. Hof and Baltic (1990, 1991) and Hof and Pickens (1986, 1987) have shown how a multilevel approach may be used to develop technically efficient regional production possibilities across a range of natural resources. Their approach utilizes a discrete set of management alternatives developed by planners at the forest level using FORPLAN, a linear programming-based forest planning model.[2] Thus, a bottom-up approach is used to generate alternatives which are then examined at the regional (or top) level of the hierarchy. Their results demonstrate that optimization at the regional level produces cost savings of as much as 11% when compared to the aggregation of individual forest-based planning alternatives. Furthermore, substantially different output and budget allocations for each forest may result from this technically more efficient perspective.

The objective of this paper is to build upon these works by extending the analysis to include: (1) multiple criteria to more appropriately reflect the multiple resource nature of natural resource management, and (2) the concept of fuzzy programming to reflect the uncertainty, vagueness, or ambiguity associated with the optimization of such large-scale systems. The California Region (Region Five) of the USDA Forest Service is used as an illustrative case study to demonstrate the consequences and implications of this approach.

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[2]See Kent et al. (1991) for additional information on FORPLAN.

MODEL FORMULATION

Fuzzy approaches to forest management planning previously have been described by Hof et al. (1986), Pickens et al. (1987), Duckstein et al. (1988), Mendoza and Sprouse (1989), Pickens and Hof (1991), Yang and Lin (1991), Bare and Mendoza (1992), Mendoza et al. (1993), and Bevers et al. (1993). Multiple objective mathematical programming applications to forest management problems largely have been limited to either goal programming or multiple objective linear programming. Additional detail about these applications can be found in Bare and Mendoza (1988), Mendoza et al. (1987), or Howard (1991). In this paper, a fuzzy approach is applied to a regional multi-resource planning problem where tradeoffs between forests within the region can be examined in the context of multiple objectives. This extends the work cited above in a direction not previously reported.

The regional model developed below is a direct outgrowth of the model previously described by Baltic and Hof (1988) and Hof and Baltic (1990, 1991).[3] Their model objective was to minimize the cost of regional forest production over five decades such that multiple resource output targets were achieved. For each forest within the region, a discrete set of management alternatives, developed using FORPLAN, were treated as the choice variables.

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[3]The authors are indebted to Dr. John G. Hof, USDA Forest Service, Fort Collins, CO for providing the data for the California Region used in this paper.

Regional resource target levels were established by aggregating the <u>preferred</u> alternatives identified in each forest's plan over all forests in the region. By constraining the model to select only one management alternative per forest, the model was cast as an integer linear programming problem. However, when this constraint was later relaxed to allow more than one management alternative to be selected, a standard linear programming formulation resulted.[4] The regional model can be written as:

$$Min \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{t=1}^{5} C_{ijt}X_{ij} \tag{1}$$

subject to

$$\sum_{j=1}^{n}\sum_{i=1}^{m} P_{ijpt}X_{ij} - T_{pt} = 0 \;\forall\; p,t \tag{2}$$

$$\sum_{i=1}^{m} X_{ij} = 1 \;\forall\; j \;\; (0,1) \tag{3}$$

$$T_{pt} \geq K_{pt} \;\forall\; p,t \tag{4}$$

$$X_{ij} \geq 0 \;\forall\; i,j \tag{5}$$

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[4]If the decision variables are continuous on the interval (0,1) then the solution may include partial selections of alternatives for a forest. Such a solution is a convex combination of the original forest alternatives and always yields a feasible but generally a sub-optimal solution (Hof and Baltic, 1991).

where

i = management alternative from a forest plan

j = forest identifier

t = 10-year time period

p = a product output from a forest plan

m = the number of management alternatives from a given forest

n = the number of forests

X_{ij} = management alternative i from forest j

P_{ijpt} = output of product p, time period t, management alternative i from forest j

C_{ijt} = cost of management alternative i from forest j in time period t

T_{pt} = a transfer variable to aggregate output of product p for time period t

K_{pt} = production targets for aggregate output of product p for time period t

Note that in the LP version of the model, the X_{ij} variables are continuous in the interval (0,1). In the case study results presented below, the analysis covers five 10-year periods; fifteen national forests in Region Five; seven resource outputs, and between 6-11 management alternatives per forest. The resource outputs are measured as average annual figures and include: (1) thousands of visitor days of dispersed recreation including wilderness and wildlife and fishing user days, (2) acres of wildlife habitat improvement including game, nongame, and endangered species, (3) size of deer population (either potential or projected) required to maintain viable populations (used as an indicator species to represent all native vertebrates), (4) thousands of pounds of fish in streams, non-wilderness fish in streams, catchable trout in streams, anadromous fish, etc., (5) projected or permitted grazing use in thousands of animal unit months, (6) timber output in millions of cubic feet, and (7) water yield in thousands of acre-feet. Costs are measured in millions of 1978 dollars and include operating, maintenance, and investment costs, timber purchaser road credits and forest fire fighting funds.

As previously shown by Hof and Baltic (1990), the LP version of this model (for Region Five) produced a cost savings of 10.8% when compared with the cost of implementing the preferred alternative as identified by each individual forest in the region. In 1978 dollars, this was a savings of $217 MM over 50 years and produced as much or more output for the seven resources over the five time periods when compared with the aggregation of preferred forest plan alternatives. In making these runs, the aggregate production targets for eq.(4) were set at the level of output that would be realized if the preferred alternative from each individual forest in the region was implemented. Specifically, for the thirty-five resource/time period possibilities, the regional LP model produced higher output levels in twenty four cases and equal outputs in the remaining eleven cases.

Some portion of the cost savings can be attributed to the fact that the regional LP solution lead to non-integer choices for seven of the fifteen forests whereas the individual forest-based preferred alternatives lead to an all-integer solution. To examine this, the X_{ij} variables were constrained to be integer on the interval (0,1), and the problem was re-solved. Convergence was slow, and when the algorithm was stopped, the best integer LP solution found (not necessarily optimal) produced a cost savings of $173 MM over 50 years. Thus, substantial cost savings are possible in either the integer or continuous variable case.

FUZZY MODEL FORMULATION

To examine the potential usefulness of a fuzzy approach, the above described model is expressed as a fuzzy linear programming (FLP) model. The essential motivation behind FLP is to view the constraints as soft (fuzzy) implying that we wish to satisfy them as closely as possible, without requiring that they be satisfied in a strict sense. And, for fuzzy objective functions, we wish to achieve an acceptable level of performance, but not necessarily the maximum (minimum) value that is possible. Thus, FLP represents multiple objectives as goals with given aspiration levels. If the problem is linear and if linear membership functions are introduced to measure both the degree of constraint violation and the degree of goal achievement, the problem can be solved using standard LP algorithms (Zimmermann, 1987). In this case, the objective is to find the maximizing solution which minimizes deviations from: (a) the goal aspiration levels associated with each fuzzy objective, and (b) the amount of constraint violation for each fuzzy constraint. Thus, as shown below, the problem is expressed as a MAXMIN problem.

A linear membership function implies that the decision maker is most satisfied when the goals or constraints are strictly satisfied and completely unsatisfied when either the minimum acceptable goal levels are not reached or a solution pushes constraint violation beyond a tolerable limit. In between these two extremes, the degree of satisfaction changes in a linear fashion. As shown by Zimmermann (1987) and Yang et al. (1991), this relationship is represented by the linear membership function $\mu_k(z_k)$ which, for fuzzy goals, is written as:

$$u_k(z_k) = \begin{cases} 1 & \text{for } z_k \geq z^o_k \\ f_k(z_k) & \text{for } z'_k \leq z_k < z^o_k, \text{ for } k = 1,2,\ldots K \\ 0 & \text{for } z_k < z'_k \end{cases} \quad (6)$$

where

$f_k(z_k)$ = $1 - [z^o_k - z_k]/[z^o_k - z'_k]$

z_k = actual achievement level of k^{th} goal

z^o_k = maximum desired level of k^{th} goal

z'_k = minimum acceptable level of k^{th} goal

As shown by Zimmermann (1976) and Bare and Mendoza (1992), under the conditions of linearity as stated above, the FLP formulation of the regional model presented earlier is:

$$Max\ \lambda \tag{7}$$

subject to

$$T_{pt} - \theta\lambda \geq I_{pt} - \theta \ \forall \ p,t \tag{8}$$

$$\sum_{j=1}^{n}\sum_{i=1}^{m} C_{ijt}X_{ij} + \theta\lambda \leq I_{ct} \ \ \forall \ t \tag{9}$$

$$0 \leq \lambda \leq 1 \tag{10}$$

Eq.(8) represents fuzzy goals for the seven resource outputs and eq.(9) represents cost. In addition to these fuzzy goals, we retain eqs.(2,3,5) from the previous model as "crisp" constraints since they must be strictly satisfied. Thus, our FLP formulation of the regional natural resource optimization problem involves fuzzy goals and crisp constraints.

Several terms in eqs.(7-10) require further explanation. First, in the FLP model, T_{pt} represent transfer variables which measure the amount of the p^{th} natural resource produced in the t^{th} time period. Second, I_{pt} represents a fixed output target for the p^{th} natural resource in the t^{th} time period. In the sample runs shown below, I_{pt} is set equal to the resource output produced by the regional LP run. Hence, for a given FLP model, the I_{pt} are constants. Third, θ represents the maximum amount of goal tolerance permitted. In the sample runs discussed below, θ is set equal to some percent of I_{pt} (e.g., $\theta = .10*I_{pt}$). Thus, the fuzzy goals represented by eq.(8) imply that we wish to produce as much of each resource as was achieved in the regional LP model, but a 10% reduction in output will be tolerated. If λ-1 , $T_{pt} = I_{pt}$, but if λ-0 , T_{pt} must be no less than 10% of what it was in the regional LP model.

Eq.(9) represents the fuzzy cost goal. Here, I_{ct} is a constant for any given run of the FLP model and is set equal to the cost for the t^{th} time period as produced by the regional LP run. The interpretation of eq.(9) is that we wish to reduce costs by some amount (e.g., $\theta = .10*I_{ct}$), but will tolerate the same cost as obtained from the regional LP run. Taken in tandem with eq.(8), it is clear that this forces a tradeoff between cost minimization and natural resource output maximization.

Lastly, as described by Zimmermann (1987), introduction of the variable λ provides a convenient way to generate the maximizing solution by solving a standard LP problem.

FUZZY MODEL SOLUTION

The FLP model shown in eqs.(7-10) was applied to the Region Five data previously described. The values for I_{pt} and I_{ct} were taken from the results of the regional LP model run. θ was set equal to 10%

of I_{pt} and I_{ct}.[5] The results of the FLP produced a solution which reduced costs over the five decades by 6.1% as compared with the regional LP solution. The maximum value of λ was 0.569. For a total cost of $1.674 billion, the maximum resource output reduction in any given time period over the five decades, was 4.3%. Of the thirty-five resource output/time period combinations, thirty-three declined an average of 3% when compared with the LP solution. The remaining two combinations increased. Thus, the FLP solution shows that if an average 3% reduction in outputs is acceptable, total costs can be reduced by as much as 6%.

A comparison of the regional LP, FLP and the individual forest-based preferred alternative solutions is shown in Table 1. The FLP solution results in more "alternative splitting" than the LP solution. In the LP solution seven forests had more than one alternative selected whereas in the FLP solution the number increased to ten forests. However, in terms of the total number of alternatives selected the FLP solution had 26 as compared to 25 for the LP solution. Splitting alternatives may be undesirable if a single alternative must be selected for a given forest. The two solutions do not appear to be drastically different when viewed from a decision space point of view, but there is some realignment of alternatives for the different forests. While not measured here, it is possible that these "alternative shifts" might not be

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[5]The maximum amount of goal tolerance can be set at any value deemed appropriate by the decision maker. We selected 10% purely for illustrative purposes.

acceptable at the individual forest-level. As previously mentioned, the reduction in total cost over the five decades of $107 million (6.1%) is accompanied by a reduction of outputs for most time periods and resources. Average reductions over fifty years are: timber (4.3%), grazing (4.1%), dispersed recreation, deer and fish populations (3.5%), wildlife habitat improvement (1.5%), and water yield (0.1%). For a given resource, these reductions are very consistent from one time period to the next.

To further demonstrate the potential usefulness of FLP, the regional LP model (eqs.(1-5)) was re-solved with the added constraints that total cost could not be less than $1.674 billion and that resource outputs for each time period could drop no more than 10% from the outputs produced by the original regional LP model. These were the conditions in effect for the FLP run the results of which were discussed above. The results of this modified regional LP model were surprising in that for the same total cost of $1.674 billion, all resource outputs <u>except</u> for water yield in decades 1, 3, and 5 were at lower levels than for comparable resources and time periods produced by the FLP run. For all intents and purposes, the FLP solution dominates the modified regional LP solution as the increases in water yield were marginal while the reductions in the other resource levels were quite substantial. This demonstrates that LP and FLP solutions can differ in decision space although producing the same objective function value.

Further, it shows that FLP may produce solutions that are more preferred by a decision maker.

Next, an LP model which minimized total cost while requiring all resource outputs for each time period to drop no more than 10% from the outputs produced by the original regional LP model was run. For this case, the total cost drops to $1.584 billion, but thirty-four of the thirty-five resource output/time period combinations decrease in comparison with the original FLP run. Total timber output over five decades declines to 1527 million cubic feet/year, just in excess of the minimum requirement of 1519.

To further examine model behavior, another set of runs was made wherein the FLP model was modified to permit up to a 50% reduction in timber output for each of five decades. For all other resource goals, including the cost goal, θ (the maximum goal tolerance) remains at 10% of the regional LP model output. Results from this model indicate that the maximum value of λ was 0.728 for a total cost of $1.645 billion -- a 1.7% decrease from the original FLP model. The output of timber declined from 1615 million cubic feet/year to 1465 -- a 9.3% decline. Of the remaining thirty resource output/time period combinations, twenty-three increased and seven decreased. Thus, very little cost savings is realized, as the reduction in timber output is offset by expenditures to increase the production of other resource outputs.

As before, the regional LP model (eqs.(1-5)) was re-solved with the added constraints that total cost could not be less than $1.645 billion and that resource outputs (excluding timber) for each time period could drop no more than 10% from the outputs produced by the original regional LP model. For timber, a 50% reduction in output was specified. These were the conditions in effect for the modified FLP run the results of which were discussed above. The results of this modified regional LP model were also surprising in that for the same total cost of $1.645 billion, all resource outputs were at lower levels than for comparable resources and time periods produced by the modified FLP run. Thus, a decision maker would likely prefer the modified FLP formulation relative to the LP formulation.

CONCLUSIONS

Optimizing natural resource management plans from a regional perspective produces more efficient solutions than those achieved by amalgamating preferred alternatives based on individual forest optimization. Using Region Five of the USDA Forest Service as a test case, Hof and Baltic (1990) have shown that an 11% cost reduction over a fifty year planning horizon is possible. Using the regional LP model formulated in eqs.(1-5), this result was verified.

To incorporate the fuzzy and multiple objective nature of regional natural resource management, this regional LP model was reformulated as a FLP. First, the FLP shown in eqs.(7-10) was formulated for all seven goal equations. The solution to the FLP model demonstrated that a 6% reduction in costs could be achieved at the expense of a 3% reduction in natural resource outputs. The FLP solution also lead to more "alternative splitting" when compared with the regional LP solution.

The regional LP model was reformulated with the added constraints that total cost could not be less than the total cost associated with the FLP solution and that resource outputs were subject to the same limitations as in the FLP model. The results of this modified LP model were inferior to those of the FLP model demonstrating the usefulness of the latter to a decision maker. These same conclusions were reached when the timber output targets were allowed to decline by up to 50% of the value shown by the regional LP solution.

Based upon our experiences, it is safe to conclude that the solutions obtained using fuzzy approaches will generally differ from those obtained by using standard LP methods. Thus, decision makers are exposed to a broader array of alternative solutions which appear to be superior to those produced by LP models. However, the degree of difference between these solutions may not be predictable *a priori*. And, differences always may not be significant. Furthermore, the use of a fuzzy approach allows one to explore a range of tradeoffs between resources which may lead to solutions which are superior to those generated by LP

Future areas for potential research are to substitute non-linear membership functions for the linear types assumed throughout this report. We have investigated this using a piecewise linear programming approximation but preliminary analysis did not produce dramatically different results from those reported herein. Another avenue to explore involves the issue of equity between forests. In essence, the preferred forest-based alternatives are developed at the forest level. Yet, the regional-based analysis ignores local preferences in lieu of reaching regionally-based solutions. Thus, the question becomes, "How much weight should be placed on the preferred alternative when undertaking the regional analysis?" Efforts are in progress to explore this issue. Lastly, the whole issue of adopting fuzzy approaches to "modelling to generate alternatives" could be explored in the context of regional natural resource planning (e.g., see Mendoza and Sprouse (1989)).

Table 1. Comparison of Regional Linear and Fuzzy Linear Programming Solutions with the Preferred Alternatives Based on Individual Forest Optimization.

Linear Programming Solution		Fuzzy Linear Programming Solution[6]
Min Cost = $1.781 billion		Min Cost = $1.674 billion
Forest	Alternatives[7]	Alternatives
Angeles	8	5(.19),8(.81)
Cleveland	3	3(.86),6(.14)
Eldorado	8	8
Inyo	2(.27),4(.73)	2(.65),6(.35)
Lassen	1(.08),2(.21) 3(.40),6(.31)	3(.28),6(.43) 9(.29)
Los Padres	9	9
Mendocino	5(.33),6(.67)	6(.47),8(.53)
Plumas	3	2(.04),3(.96)
San Bernardino	2	2
Sequoia	5	2(.49),5(.51)
Shasta-Trinity	2(.07),5(.93)	2(.45),5(.55)
Sierra	2	4(.84),5(.16)
Six Rivers	1(.55),4(.18) 8(.27)	8
Stanislaus	2(.89),7(.11)	2
Tahoe	4(.73),8(.27)	4(.89),7(.11)

Individual Forest-Based Optimization: Min Cost = $1.998 billion

For all forests the preferred alternative was alternative one.

[6]For the case where the goal tolerance was 10% of the regional LP model resource outputs.

[7]Numbers in parentheses refer to the proportion of the forest alternative selected in the optimal solution.

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